

The Cosmological QCD Phase Transition Revisited

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Abstract

The QCD phase diagram might exhibit a first order phase transition for large baryochemical potentials. We explore the cosmological implications of such a QCD phase transition in the early universe. We propose that the large baryon-asymmetry is diluted by a little inflation where the universe is trapped in a false vacuum state of QCD. The little inflation is stopped by bubble nucleation which leads to primordial production of the seeds of extragalactic magnetic fields, primordial black holes and gravitational waves. In addition the power spectrum of cold dark matter can be affected up to mass scales of $10^9 M_\odot$. The imprints of the cosmological QCD phase transition on the gravitational wave background can be explored with the future gravitational wave detectors LISA and BBO and with pulsar timing.

1 Introduction

The history of the early universe left an imprint on the presently observed cosmos. According to the Friedmann equations the temperature increases inversely proportional with scale parameter a so that the early universe passes through the big bang nucleosynthesis (BBN) at $t = 1$ s to 3 minutes (corresponding to $T = 0.1$ to 1 MeV), the QCD phase transition at $t \approx 10^{-5}$ s ($T \approx 150$ MeV) and the electroweak phase transition at $t \approx 10^{-10}$ s ($T \approx 100$ GeV). BBN and the cosmological electroweak phase transition have received considerable attention in the last years by studying the production of light elements and baryogenesis. On the other hand the cosmological QCD phase transition does not seem to be associated with a key observable in today's universe. In this contribution we revisit the cosmological QCD phase transition and discuss some new cosmological signals in view of the little inflation scenario proposed by us [1, 2]. The basic ingredients for a short inflationary period are an Affleck-Dine-type baryogenesis, in order to achieve a large initial baryon-to-photon ratio, and a metastable vacuum due to the nonvanishing vacuum expectation values of QCD at high net baryon densities.

The implications and possible signals of such a scenario are surprisingly rich and involve large-scale structure formation up to dwarf galaxy scales, the production of dark matter (WIMPs) and mini black holes, provide possibly the seeds of the cosmological magnetic fields and leave an imprint on the gravitational wave background. The latter signal is particularly interesting, as it can be probed with the gravitational wave detectors LISA and BBO and with pulsar timing. We point out that the QCD phase transition without inflation has been widely discussed in the literature before although assuming a first order phase transition at nearly vanishing baryon densities (see e.g. [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]).

and [14] for a review) so that these results can be transferred to the little inflation scenario presented in the following. The concept of a mini-inflation (or tepid inflation) at the QCD phase transition has been also introduced earlier by Kämpfer et al. [15, 16, 17, 18] and for a substantially stronger inflationary period by Borghini et al. [19] which in both cases will dilute an initial high net baryon density to the presently observed small value of the baryon-to-photon ratio.

2 A Little Inflation at the QCD Phase Transition

In the standard cosmology the early universe passes through the QCD phase diagram at small baryon density and high temperature, a region which is presently probed with ultrarelativistic heavy-ion collisions at BNL's RHIC and CERN's LHC. The other part of the QCD phase diagram, high baryon densities and moderate to low temperatures, is the realm of core-collapse supernovae and neutron stars and will be investigated in more detail in terrestrial experiments with the upcoming Compressed Baryonic Experiment CBM at GSI's Facility for Antiproton and Ion Research FAIR.

From the analysis of the microwave background radiation and big bang nucleosynthesis one deduces a baryon-to-photon ratio of $n_B/s \sim n_B/n_\gamma \sim \mu/T \sim 10^{-9}$, a ratio which is conserved after baryogenesis and the last first order non-equilibrium phase transition in the early universe as entropy is conserved during the standard cosmological evolution. Hence, the early universe evolves along $\mu/T \sim 10^{-9} \sim 0$ in the QCD phase diagram. As lattice data indicates a crossover transition at vanishing baryochemical potential [20], nothing spectacular is happening and no particular strong cosmological signals are expected from the QCD phase transition.

The Friedmann equation for a radiation dominated universe reads

$$H^2 = \frac{8\pi G}{3} \rho \sim g(T) \frac{T^4}{M_p^2}$$

where $g(T)$ is the effective number of relativistic degrees of freedom at the temperature T . The Hubble time is related to the true time by $t = 3t_H$ for a radiation dominated universe and is given by

$$t_H = \frac{1}{H} \sim g^{-1/2} \frac{M_P}{T^2} \implies \frac{t}{1 \text{ sec}} \sim \left(\frac{1 \text{ MeV}}{T} \right)^2$$

So for the QCD phase transition at $T = T_c \sim 150 \text{ MeV}$ one arrives at a time of about 10^{-5} s after the big bang.

We explore in the following the scenario that the early universe passed through a first order QCD phase transition which is presently advocated for large baryochemical potentials. First, we address the question whether this is possible or not with the present cosmological data. Second, we delineate possible cosmological signals of a first order QCD phase transition which could be observable in the future.

For a first order phase transition there exists a false metastable vacuum, in which the universe could be trapped during cooldown. As the total energy density is then determined by the constant potential of this false vacuum state, the universe evolves according to de Sitter solution. One can easily derive from the Friedmann equation that a constant energy density implies an exponential expansion rate

$$H = \dot{a}/a \sim M_p^{-1} \rho_v^{1/2} = H_v = \text{const.} \rightarrow a \sim \exp(H_v \cdot t)$$

so that the universe goes through another inflationary epoch. During this time the universe supercools and the density decreases exponentially while the ratio μ/T is preserved. At the end of inflation, the universe falls into the true vacuum state which releases latent heat. The universe gets reheated with a final temperature similar to the one at the start of inflation. The parameters are such that the

temperature and baryochemical potential are now below the phase transition line. The corresponding increase in photon density results in a reduced baryon-to-photon ratio. For our purposes just a few e-folds are enough to dilute the baryon to photon ratio to the presently observed value of about 10^{-9} . Then for a gas of massless quarks, the baryon-to-photon ratio before and after inflation are related by:

$$\left(\frac{\mu}{T}\right)_f \approx \left(\frac{a_i}{a_f}\right)^3 \left(\frac{\mu}{T}\right)_i$$

as the baryon density is diluted by the scale factor cubed. Hence an initial ratio of $(\mu/T)_i \sim \mathcal{O}(1)$ can be reached for just $N = \ln(a_f/a_i) \sim \ln(10^3) \sim 7$ e-folds (for comparison standard inflation at the GUT scale needs $N \sim 50$ e-folds). For such high values of the baryon-to-photon ratio a first order QCD phase transition is presently discussed in the literature in connection with chiral symmetry restoration. A first order QCD phase transition in the early universe could have also been triggered by a large lepton asymmetry as pointed out recently by Schwarz and Stuke [21] (for a popular account on the “bubbling universe” at the QCD phase transition see [22]). We note in passing that the high baryon asymmetry in our scenario presumably also implies a correspondingly high lepton asymmetry. A first-order phase transition in QCD can be modeled within the linear σ model [23] with the possibility of a quench at finite chemical potentials [24]. The quark condensate or here the expectation value of the σ -field serves as an order parameter. Actually, the QCD phase transition might be more complex owing to its nontrivial vacuum structure. There is a second order parameter, the Polyakov loop, which is related to the gauge sector of the theory and its deconfinement phase transition. The trace anomaly of QCD can be related to the vacuum expectation value of the gluon condensate and its scaling properties. So it might well be that there is a second scalar field, a dilaton field, which has to be taken into account in an effective description of the phase transition. Interestingly, such an effective model with two scalar fields opens the door for a hybrid inflation scenario in cosmology within the standard model.

A little inflation in the QCD phase diagram starts with a high initial value of baryon-to-photon ratio which for massless particles is proportional to the ratio of baryochemical potential to temperature so that $\mu/T \sim 1$. Such a large value can be inherited from some earlier nonequilibrium processes producing a net baryon number as in Affleck-Dine baryogenesis [25]. Then the conditions in the early universe are such that the first order phase transition line at large baryochemical potentials of QCD is hit and the universe is trapped in false vacuum state. The constant and nonvanishing vacuum energy density leads to an inflationary expansion associated with an exponential supercooling and dilution of matter. Note that during this evolution the universe is not in an equilibrated state while both the temperature and the baryochemical potential drop exponentially with $\mu/T = \text{const.}$. At low temperatures the barrier to the true vacuum state becomes so low that the universe rolls down to the true vacuum state. The released latent heat is converted to particle production and eventually reheats the universe to temperatures of $T \sim T_c$. During reheating the baryon number is conserved so that the net baryon density is still substantially reduced compared to the initial state before inflation, so that $\mu/T \sim 10^{-9}$, the value observed today. Afterwards the standard cosmological evolution follows proceeding to big bang nucleosynthesis.

The evolution of energy densities during the little inflation period can be discussed in quite general terms. The energy density of nonrelativistic dark matter falls off as a^{-3} always. The total energy density, however, is determined by the one of the QCD vacuum once the universe is trapped in the false high-temperature QCD vacuum state and inflation starts. The radiation energy density falls off initially as a^{-4} until the end of inflation. Then energy is gained from the phase transition from the false vacuum state to the true one which is about the energy scale of the QCD vacuum. This energy is transferred to heat and the production of relativistic particles which thereby increases the radiation energy density correspondingly. The baryon density and dark matter density to entropy ratio will be diluted at the end of inflation due to the production of entropy. By construction the maximum length

of inflation is about the one given by the initial value of the baryon to entropy ratio which translates to a ratio of the initial and final value of the scale parameter of about $a_f/a_i = 10^3$.

3 Cosmological implications of the QCD phase transition

A prominent candidate for cold dark matter are weakly interacting massive particles (WIMPs), which freeze-out while being non-relativistic with an annihilation cross section similar to the one expected from weak interactions between e.g. SUSY particles. The present day energy density for WIMPs is just given by the annihilation cross section with logarithmic corrections from the mass of the WIMP as $\Omega_{CDM} \sim \sigma_{\text{weak}}/\sigma_{\text{ann.}}$. If the number density of dark matter is diluted by a factor $(a_f/a_i)^3 \sim 10^9$ due to the little inflation, the annihilation cross section has to be reduced accordingly so that the present day abundance is matched despite the dilution factor. Then the production cross section is reduced by a similar factor. The observation of the dark matter WIMP at the LHC would then not be possible in the little inflation scenario.

The nonstandard evolution of the energy densities will leave an imprint on the power spectrum of dark matter. The dark matter mass within the horizon at the critical temperature of QCD of $T_c \approx 150$ MeV is just $10^{-9}M_\odot$, too small a mass scale to have any cosmological consequences. However, the mass scale is boosted by the little inflation by at least a factor $(a_f/a_i)^3 \approx 10^9$ so that mass scales of up to $1M_\odot$ are affected. There is an additional effect for modes $k_{ph} < H$ at the beginning of inflation as there are two scales involved during inflation: one is the Hubble parameter $H \propto \rho_v^{1/2} \sim \text{const.}$ and the other one is

$$\dot{H} = -4\pi G(\rho + p) = -4\pi G(\rho_{dm} + 4\rho_r/3) \propto (a_i/a)^q$$

where $q = 3 \dots 4$ depending on whether the matter or radiation energy density is the dominant contribution. Note, that the total energy density is given by the constant vacuum energy density which, however, drops out for \dot{H} . For standard inflation only the former one, the Hubble horizon, is important but for a small period of inflation also the latter scale leaves an observable effect on the power spectrum. Correspondingly, there are three different spectral regimes to be considered:

- $(k_{ph}/H)_i > a_f/a_i$: always subhubble
- $a_f/a_i > (k_{ph}/H)_i > (a_i/a_f)^{q/2}$: intermediate
- $(k_{ph}/H)_i < (a_i/a_f)^{q/2}$: unaffected

Hence, the highest mass scale affected can be as large as $M_{max} \sim 10^{-9}M_\odot(a_f/a_i)^{3q/2} \sim (10^{4.5} - 10^9)M_\odot$ depending on the value of q . For interesting inflation lengths the value of q will be closer to 3 than to 4, because dark matter will then be more abundant than radiation for most of the inflationary phase, resulting in a maximum mass of $\sim 10^6M_\odot$. These mass scales are of cosmological interest as it reaches the mass scales of globular clusters and dwarf galaxies. The power spectrum will be reduced, so that it would be interesting to study its impact on the cuspy core density distribution of dark matter in small galaxies and the large number of halo structures seen in standard structure formation.

The nonequilibrium process ending inflation is likely to produce bubbles by nucleation or by spinodal decomposition. The fluctuations in density together with a softening of the equation of state at the phase transition can create primordial black holes by collapsing bubbles [8, 26]. The maximum mass of the black hole is given by the total enclosed energy density at the time of the QCD phase transition, i.e. after inflation, which is about $M_{bh} \sim M_{hubble} \sim 1M_\odot$.

The nonequilibrium phase transition and the formation of bubbles will generate perturbations in the metric. It turns out that the tensor perturbations are directly related to the QCD trace anomaly. The

equation of motion for the tensor perturbation amplitude $v_k = a \cdot h_k$ in Fourier space (gauge invariant) is given in a gauge invariant way by:

$$v_k''(\eta) + \left(k^2 - \frac{a''}{a}\right) v_k(\eta) = 0 \quad \text{with} \quad \frac{a''}{a} = \frac{4\pi G a^2}{3} (\rho - 3p)$$

Hence, only the trace of the energy-momentum tensor is responsible for generating tensor perturbation, i.e. gravitational waves from the phase transition. We use several parameterizations of lattice gauge calculations as input taken from [27, 28] and compare with the simple bag model.

The energy density in the gravitational wave background is usually expressed for a given wavenumber k and reads $\Omega_g(k) = \frac{1}{\rho_c} \frac{d\rho_g}{d \ln k}$. After horizon entry, the mode h_k is damped by the scale factor as $1/a$. Using entropy conservation and the Friedmann equations one can show that the energy density will exhibit a steplike change due to the different relativistic degrees of freedom before and after the phase transition as

$$\frac{\Omega_g(\nu \gg \nu^*)}{\Omega_g(\nu \ll \nu^*)} = \left(\frac{g_f}{g_i}\right)^{1/3} \sim 0.7$$

which has been shown by Schwarz [29]. The characteristic frequency for the QCD phase transition is redshifted today by

$$\nu_{\text{peak}} \sim H_c \cdot T_{\gamma,0}/T_c \sim T_c/M_p \cdot T_{\gamma,0} \sim 10^{-8} \text{ Hz}$$

The absolute maximum amplitude one can expect is $h \sim a/a_0 \sim 10^{-12}$.

We calculated the gravitational wave background normalized to the one for low frequencies for the standard cosmological evolution [2]. We find that a step in the gravitational wave background around $\nu \sim 10^{-8} \text{ Hz}$ is clearly seen resulting from the QCD phase transition which is about 0.7. The results are rather insensitive to the parameterizations used from lattice gauge calculations and the details of the phase transition. There appears some more pronounced oscillations for the MIT bag model parameterization due to the sharper drop in energy density compared to the lattice data.

The spectrum of gravitational waves for a little inflation will be drastically different, as the amplitudes are exponentially suppressed during inflation: $h \sim 1/a \sim \exp(H \cdot t)$. The gravitational wave background will drop as ν^{-4} so that the high frequency part will be a factor 10^{12} smaller compared to the one for standard cosmology.

Gravitational waves can be presently measured with ground based interferometers as LIGO, GEO600, TAMA, and VIRGO, which, however, are mostly sensitive to frequency of about 100–1000 Hz. The future space-based interferometer LISA will explore the gravitational wave band around a frequency of 10^{-5} Hz . Interestingly, gravitational waves can be also detected by combining the timing signal for several millisecond pulsars. First limits for the gravitational wave background have been already set by Parkes Pulsar Timing Array [30]. The predicted step frequency in the amplitude is close to highest sensitivity of pulsar timing. In the future the Square Kilometer Array SKA will increase the sensitivity in this frequency range by an order of magnitude by measuring hundreds of millisecond pulsars. The Big Bang Observer BBO, a planned NASA mission, is aimed at measuring the gravitational wave background from inflation in a frequency band between the ones of LISA and LIGO (see e.g. [31]). In the little inflation scenario, the gravitational wave background can be detected by looking at the polarization pattern at the CMB but would be unmeasurably small for the frequency scales probed by BBO.

Gravitational waves can be generated from bubble collisions at the end of the little inflation period in addition to the gravitational wave background. The gravitational wave amplitude scales then as $h(\nu) \propto \nu^{-1/2}$ for $\nu < H$ which is just white noise, while it scales as $h(\nu) \propto \nu^{-2 \dots -1}$ for the higher frequencies within the Hubble horizon $\nu > H$ depending on whether multi-bubble collisions are taken into account or not [6, 11, 13]. For the flatter spectrum, the produced gravitational waves could be measured with LISA [2].

4 Summary

We have explored a cosmological scenario where the early universe passes through a first order QCD phase transition. For large initial net baryon numbers so that $\mu/T \sim \mathcal{O}(\infty)$ there might be a first order phase transition line in the QCD phase diagram as suggested by effective models of QCD. The universe is trapped in a metastable false vacuum state generating a little inflation with about seven e-folds. This little inflationary period would generate potentially observable signals.

- The power spectrum of large-scale structure is modified up to mass scales of $M \sim 10^9 M_\odot$ (without QCD inflation only mass scales up to the horizon mass $\sim 10^{-9} M_\odot$ can be affected).
- The cold dark matter density is diluted by a factor 10^{-9} so that a reduced WIMP annihilation cross section is needed as $\Omega_{\text{CDM}} \sim \sigma_{\text{weak}}/\sigma_{\text{ann}}$ with implications for the WIMP searches at the LHC.
- The first order phase transition generates seeds of (extra)galactic magnetic fields by the collisions of charged bubbles which would be a viable scenario within the standard model again (see [12]).
- The change of the scale factor will modify the gravitational wave background by suppressing it for frequencies above about 10^{-8} Hz. Colliding bubbles and turbulence can generate additional gravitational waves which can be observable with pulsar timing and eventually with LISA.

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